The Theory of Three-level Photon Echo for Quasi-degenerated Levels

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Abstract

The three-level photon echo has been described in different works[1,2] by using rotating wave approximation but none of them didnt get results which show the effects of detuning frequencies on frequency of ground level of system; also the effect distance between two generated levels on signal resonance is being neglected. In this work, we studied a -type system theoretically and numerically. By considering the Dopplers effect in environment, we get different equation for polarization of echo signal and its intensity.

1 Introduction

Quantum optical data storage is a key element in quantum information processing such as quantum computing and long-haul quantum communications based on quantum repeaters. However, most modified photon echoes protocols are still limited by lack of homogenous resonance signal to explain time scales for the different systems. Photon echo spectroscopy is the most important way which can deeply extract microscopic information about the time scales of molecular and collective dynamics of condensed phase. Photon echo spectroscopy can cover any inhomogeneous broadening in state. The three-level photon echo is made of three different signals in a medium.

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Among these levels or dots distributed homogenously resonance frequencies. In the present paper, by using rotating wave approximation, we showed that proportion of detuning frequencies is equal with proportion of frequencies between degenerated and ground levels. Our method to solve relevant equations is acceptable but simpler than the other methods. This equation can be used in order to determine the resonance form of echo signal. As has been shown in previously works, signal resonance is depending on frequencies between degenerated and ground levels. After finding the relationship between frequencies and their detuning amounts, its obvious to predict an equation for delay time as ratio of frequencies. Polarization of signal usually depends on frequencies between levels and frequencies detuning. This relationship gets an exponentially resonance for signal polarization between ground level and second degenerated level. By using this results as numerical data, we found an exponentially function for intensity as frequencies, signal delay time and distance of ground level from two degenerated levels. As has been shown previously, it is possible to write equation for frequency detuning between laser fields either as function of frequencies or as function of frequency distance between ground level and second degenerated and frequency detuning between laser field and transition frequency. In the last section of paper, the effect of frequency distance between these two levels on the general intensity of output signal has been proved. It seems that these results have strongly consistent with theoretical background and other same works. In other words, our results for the effect of frequency distance between ground and low degenerated level can give decay coefficient of signal in different cases of system and position of echo signal in the long timescale for the case of qusi-degenerated levels can be predicted.

2 Theoretical Framework

As using strength field for transition between degenerated levels and ground level, the reaction time of pulses increase which means decay of signals memory time. But this system have $\Lambda-$type structure in three-level atoms with long-life lower-levels. These lower-levels are fine-structure components which the interaction between them is so weak and is only due to spin-spin structure. This can help to create a long-living houl for using as information/signal memory. In this system we have two strong femtosecond pulses and a detuning pulse. In the real systems, delay time between detuning driving pulses is 500microsecond which compared to pulse time (100femtosecond) increases the signals memory time 10^9 times more than signals

time. In order to prove this process we studied a three-level system. The system considered a three-level $\Lambda-$type system with a ground state (2). The transition$2 \leftrightarrow 3$ is driven by frequency ω_{23} and Rabbi frequency G_c.The transition $2 \leftrightarrow 1$ is driven by frequency ω_{21} and Rabbi Frequency g_c . The rates of emission from 1 and 3 are denoted by $2\gamma_1$ and $2\gamma_2$ so detuning amounts of the probe and coupling fields are $\Delta_1 = \omega_{12} - \omega_1$ and $\Delta_2 = \omega_{23} - \omega_3$.
From the Liouville equation $\dot{\rho} = \frac{i}{\hbar}[H, \rho] - \gamma\rho$ Where H- Hamiltonian system of atom and field and ρ- element of density matrix. The matrix density equations for three-level system interacting with a pulsed laser [4, 5] in a semi classical dipole rotating-wave approximation are

$$\begin{cases} \dot{\rho}_{11} = -2\gamma_1\rho_{11} + 2\Lambda\rho_{22} - V(\rho_{31} + \rho_{13}) + ig_c\rho_{21} - ig_c^*\rho_{12} \\ \dot{\rho}_{22} = -2\gamma_2\rho_{22} - V(\rho_{31} + \rho_{13}) + iG_c\rho_{23} - iG_c^*\rho_{32} \\ \dot{\rho}_{33} = -2\gamma_3\rho_{33} + 2\Lambda\rho_{22} - V(\rho_{31} + \rho_{13}) + iG_c\rho_{12} - iG_c^*\rho_{21} \\ \dot{\rho}_{12} = -(\gamma_1 + \Lambda + i\Delta_1)\rho_{12} - V\rho_{23} + ig_p(\rho_{22} - \rho_{11}) - G_c\rho_{13} \\ \dot{\rho}_{13} = -[\gamma_1 + \gamma_3 + i(\Delta_3 - \Delta_1 - d)]\rho_{13} + ig_p\rho_{32} - iG_c^*\rho_{23} \\ \dot{\rho}_{23} = -[\gamma_2 + \Lambda - i(\Delta_3 - \Delta_2 - d)]\rho_{23} - V\rho_{13} - ig_p\rho_{21} \\ +iG_c(\rho_{33} - \rho_{22}) \end{cases} \tag{1}$$

Where parameter ρ_{ii} represented the population of i level and ρ_{ij} represented population between levels, $V = -D.E_{laser}\cos(\omega_2 t)$ is interaction of medium with laser field,D=dipole moment and d= frequency distance between two degenerated levels. We introduced the Rabi frequencies as $g_p = g_0 e^{i\Phi_p}$ and $G_c = g_0 e^{i\Phi_c}$ where is phase of frequency. There are several situations in this case:$\begin{cases} \varphi_p = \varphi_c \\ \varphi_p \neq \varphi_c \end{cases}$ In the first situation laser fields are polarized likely and

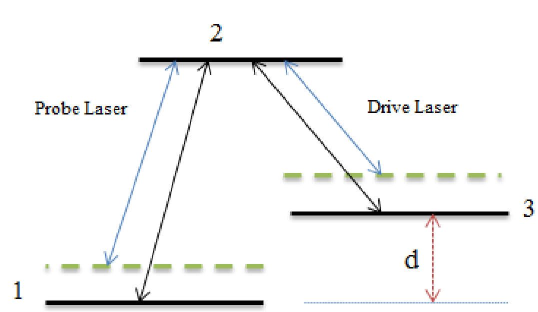

Figure 1: Λ-type system with driving and probing field

in the second they are polarized differently. In general words, can say that phases have small influence on the photon-echo signal. We calculate the time of echo for ensemble of atoms in gas; in which occurs non-uniform spread of frequencies with the Maxwell velocity distribution due to the Doppler Effect, which means proportion$\frac{\Delta_{12}}{\Delta_{23}} = \frac{\omega_{12}}{\omega_{23}}$is valid where$\Delta_{12} = \omega_{laser1} - \omega_{21}, \Delta_{23} = \omega_{laser2} - \omega_{32}$. During the first emission t_0 to t_{12} and second t_{12} to T (second pulse and echo), level 3 does not expose to field and phases of atoms in this level acquire phase shift $(T - t_0) * ({}_{12} - {}_{23})\frac{v}{c}$. But phase shift at level 2 does not occur not during first nor second emission [7]. Therefore for level 2 general phase shift during emissions is $\frac{v*\omega_{12}}{c}(T - t_{12})$. Polarization during emission from 2 $\longrightarrow$ 3 is driven as$P_{23}(t) \sim \exp[\Delta(T - t_{12}) - \Delta^{'}(T - t_0)]$ or

$$P_{23}(t) \sim \exp[(-\omega_{12}(T - t_{12}) - (\omega_{12} - \omega_{23})(T - t_0))\frac{v}{c}] \tag{2}$$

The echo signal occurs if phase treats to zero, so we can write equation for signal delay time as:$T = t_0 + \frac{\omega_{12}}{\omega_{23}} * t_{12}$. By using the elements of density matrix and solving Bloch equation for main its elements(x,y,z), equation for polarizations in x ,y and z directions can be written as $P_x = P_z = 0$. From the equation.1, its obvious that

$$\begin{cases} P_{23} = \sum\limits_i - \gamma(\rho_{23} - \rho_{13}) + i\rho_{23}(k_i\vartheta_i - k_i^{'}\vartheta_i^{'}) \\ -ig_p(\rho_{21} - \rho_{33} + \rho_{22}) \\ P_{12} = -\sum\limits_i \gamma(\rho_{12} + \rho_{23}) + ik_i\vartheta_i\rho_{12} + ig_p(\rho_{22} - \rho_{11} + i\rho_{13}) \end{cases} \tag{3}$$

Wherek_i υ_i and $k_i^{'}$ $\upsilon_i^{'}$ are the wave coefficients and frequencies of different pulses. By using Gaussian probability functions as $f(\Delta\omega t)$ for inhomogeneous distribution of velocity in system, we can write:

$$\begin{cases} P_y = \sum\limits_i P_y^i = -P_0 \int_{-\infty}^{+\infty} cos(\Delta\omega t).f(\Delta\omega t)d(\Delta\omega) \\ P_y^i = -P_0^i\{\cos(\Delta\omega_{2i}t_{12})\cos[\Delta\omega_{2i}(t - \Delta\omega_{2i}t_{12})] + \\ \qquad sin(\Delta\omega_{2i}t_{12})sin[\Delta\omega_{2i}(t - \Delta\omega_{2i}t_{12})]\} \end{cases} \tag{4}$$

In the result of this method$P_y \sim P_0 \exp[-\frac{(t - \frac{\omega_{21}}{\omega_{23}}t_{12})}{T^*}]$. Where T^* is half-width Gaussian distribution.

3 Numerical Framework

In this section, we give some numerical simulations by using RWA expression for the state population given from Sec .II. The relevant system properties are length of the signal pulses of 100 fs, the probabilities of relaxation transitions are denoted by γ_{21} and $\gamma_{23} \sim 10^{10}c^{-1}$, Homogeneous broadening $\Gamma \sim 10^{12}c^{-1}, \gamma_{13} = 0, \Gamma_{13} = 0, \Delta_{12} = 50, \frac{\Delta_{12}}{\Delta_{23}} = 1$. When the pulse resonates with this excited state, dynamics contributions of excited and ground states induce signal. The polarization spectrum of nonlinear system is found from Fourier transform of elements of density matrix.

Distance between control pulses for weak degenerate levels b =0.05, c=0.10, a=0.02(Signals polarized differently) As it has been shown in figure.3 intensity of signal is depends on the delay time exponential. This figure has been gotten by using parameter of distance in equation for detuning probe field [2]. the intensity of echo signal depends on the modulation index of process grating which led to a reduction of the echo signal.Based on [7] during delay time T, spectral diffusion cases frequency grating to earase and their magnitude to decay in time.The results of our simulation are shown in figure.2. They show a maximum in the signal intensity around T.This peak in related to coherant pump-probe signals.It's obvious that there is no fast decay for intensity after separation of third signal from beams in time.So we have good decaying behaviour on long timescales. In this figure we have periodic exponentially decay for signal intensity. The range of quantum beats depends on the frequency distance between levels (d). This phenomenon has been observed experimentally and used in echo spectroscopy to find the weak degeneracy in atoms. Difference between figures of signal intensity in two cases (case of polarization) is in place of periods. Signal in these two cases occurred likely but in different times. This difference is equal with coefficient γ. For explain this decrease, we use energy changes of a photon which adds to and removes from the beam, due to the decrease of the upper and lower level. For the number of photons N_p of the beam

$$\dot{N}_i = \frac{dN_i}{dt} = -N_i(TransitionProbability)) = -N_i B_{ij} \rho_{ij} \tag{5}$$

Where B is proportionality factor in transition probability between the energy levels (i and j) with degeneracies depend on ω: $N_{12} = \frac{\omega_2}{\omega_1} N_1 - N_2$ For our system B and are equal to 1, so we have

$$\dot{N}_i = -\frac{\omega_2}{\omega_1} \gamma_{12} \tag{6}$$

for spectral energy density:

$$\dot{\rho} = \Delta E_{12} \frac{d\dot{N}_p}{dVolume} = \Delta E_{12}(-\frac{\omega_2}{\omega_1}\gamma_{12}) \tag{7}$$

$$\rho(t) = \rho_0 \ e^{\Delta E_{12}(-\frac{\omega_2}{\omega_1}\gamma_{12})} \tag{8}$$

But $I \propto \rho$ also $I = \frac{Energy}{Areatime} = I_0 e^(- \frac{\omega_2}{\omega_1}\gamma_{12}) = I_0 e^{at}$ The position of photon echo signal is proportional to the detuning time between driving pulses. We can get a decay coefficient from this figure as $\frac{\triangle_{12}}{\triangle_{23}}\gamma_{12} = a$ which has a good agreement with theory [1].
When distance between 1 and 3 is zero means that they have the same ground level, function of I is like $\delta(t_1 2\gamma)$ - function. Intensity in this case decrease rapidly and there is only one max point in figure at first part signal. This figure is same as intensity of signal when energy of ground level is zero [18]. You can see that for $d = \infty$ fairly there are not any quantum beats and we have a very slow population relaxation compared with theoretical coefficient of signal decay. There are some remarkable characteristics of 3-level photon echo by using fluorescence excitation measurements in figures.2and 3. A direct relationship of spectral diffusion has been provided by the variation injections rate measured as a function of time between $1 \longrightarrow 2$. Different view of figure.3 shows the importance of distance between two degenerate levels on signal density.

4 Result

In this paper we developed a general theory of three-level system by assuming rotating wave approximation and gave an analytical solution for problem of frequency distance between ground and low degenerated level. Moreover in the three-level system we gave some approximate solutions which are important in order to analyze experimental data. We carried out numerical simulation of an atomic medium to the field of two driving pulse and one signal pulse in Λ-scheme. It was shown that the time of two long-wavelength three-pulse echoes is proportional with ratio of detuning of the resonance lines of transitions in a non-uniform distribution. As the result of numerical simulation for RWA, the dependence of the signal from the delay time in weak degenerate levels. These results are consistent with experiment and theory. By this time all figures and theories have neglected relationship between signals intensity and detuning amounts of fields, but in this work, we proved linear signature. In general words, photon echo spectra are found to

be strongly dependent on frequency of pump and probe frequencies of system also their dependence with detuning amounts of frequencies. During the last part of simulation, are found the importance effect of frequency distance between two degenerate levels (d) on the delay time of signal. This dependence could help us to make coherent distribution of signal in system. For example if have been investigated investigate the characteristics of a quantum dot, we find out that if first impulse has time interval of 100fs, we can increase the echo time by increasing the ratio of detuning time frequencies and frequency distance between two degenerated levels. Basic on the calculations the ratio of pulse signal to interval between 1 and 2 is $\frac{T_{pulse}}{t_{12}} \sim 10^{-6}$. In other words by using the effect of frequency distance on the time of echo signal occurrence, its possible to increase memory time of this dot to 500s. Increasing memory time of quantum dots is one of the most important tasks of quantum electronics.In order to use this method for quantum dots, signal intensity has to be as strong as to give $g_p t_{12} \sim \pi$(Impulse area). In other words, we have to collect a system from $\sim 5 \times 10^{15}$atoms.

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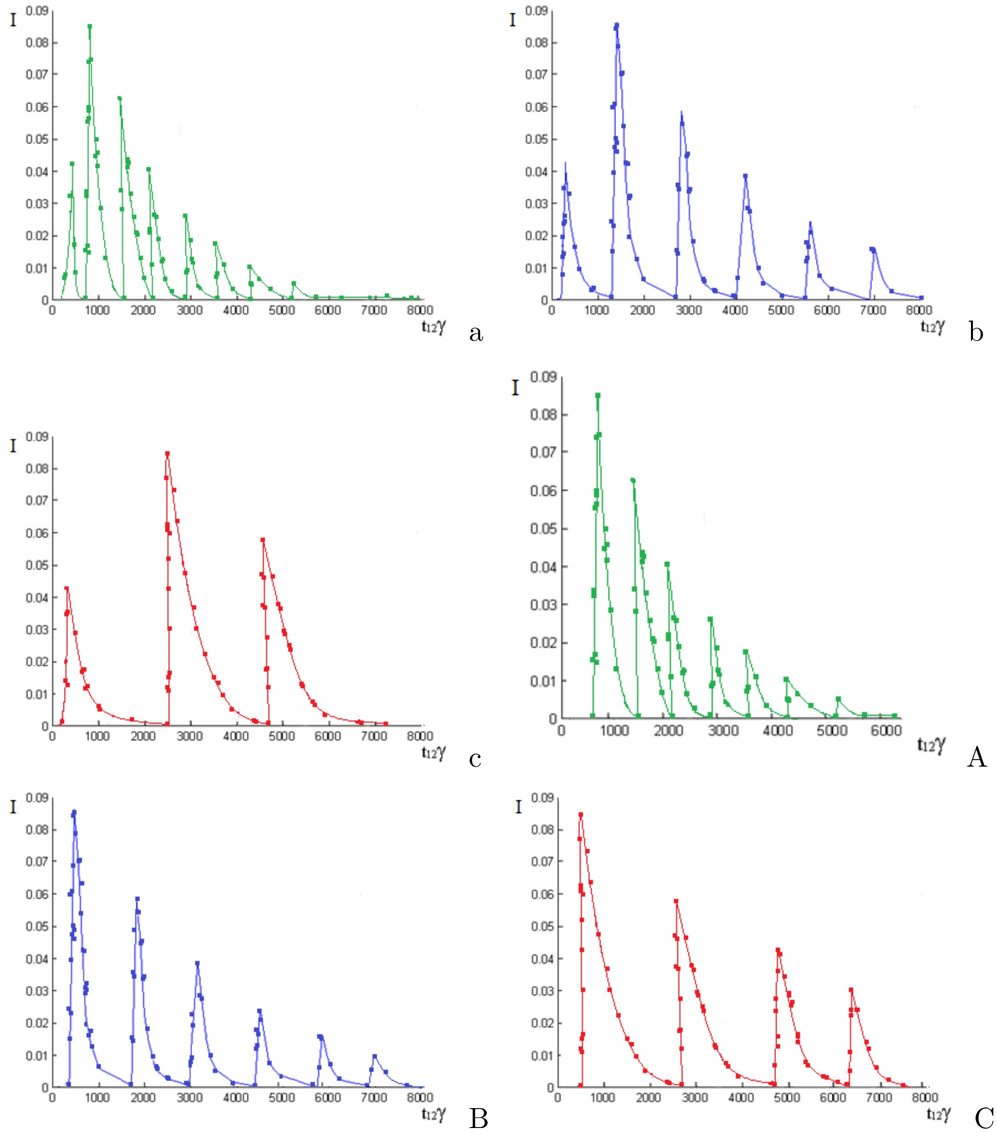

Figure 2: intensity of echo signal as a function of delay time

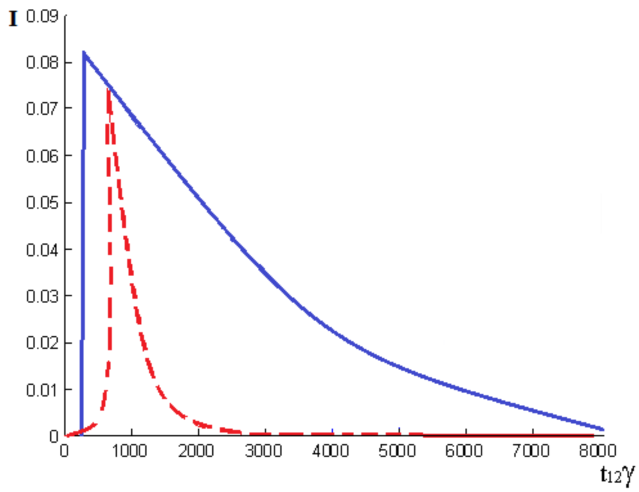

Figure 3: Intensity of echo signal as a function of $t_{12}\gamma$. For frequency distance is d=0 - (dashed), $d = \infty$- (solid)